We also have, from reciprocity,

$$A_2 F_{21} = A_1 F_{12}$$

so

$$F_{12} = \frac{A_2}{A_1} F_{21} = \frac{(0.8)(0.5)}{0.942} = 0.425 \qquad [d]$$

Combining (*b*), (*c*), and (*d*) gives

$$F_{13} = 0.686 - 0.425 = 0.261$$

Finally,

$$F_{11} = 1 - F_{12} - F_{13} = 1 - 0.425 - 0.261 = 0.314$$

This example illustrates how one may make use of clever geometric considerations to calculate the radiation shape factors.

8-6 | HEAT EXCHANGE BETWEEN NONBLACKBODIES

In addition to the assumptions stated above, we shall also assume that the radiosity and irradiation are uniform over each surface. This assumption is not strictly correct, even for ideal gray diffuse surfaces, but the problems become exceedingly complex when this analytical restriction is not imposed. Sparrow and Cess [10] give a discussion of such problems. As shown in Figure 8-24, the radiosity is the sum of the energy emitted and the energy reflected when no energy is transmitted, or

$$J = \epsilon E_b + \rho G \qquad \textbf{[8-36]}$$

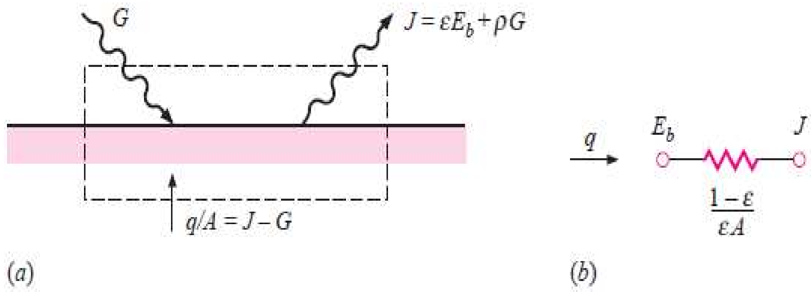
where ϵ is the emissivity and E_b is the blackbody emissive power. Since the transmissivity is assumed to be zero, the reflectivity may be expressed as

$$\rho = 1 - \alpha = 1 - \epsilon$$

so that

$$J = \epsilon E_b + (1 - \epsilon)G \qquad \textbf{[8-37]}$$

Figure 8-24 | (*a*) Surface energy balance for opaque material; (*b*) element representing "surface resistance" in the radiation-network method.



The net energy leaving the surface is the difference between the radiosity and the irradiation:

$$\frac{q}{A} = J - G = \epsilon E_b + (1 - \epsilon)G - G$$

Solving for G in terms of J from Equation (8-37),

$$q = \frac{\epsilon A}{1 - \epsilon}(E_b - J)$$

or

$$q = \frac{E_b - J}{(1 - \epsilon)/\epsilon A} \qquad \textbf{[8-38]}$$

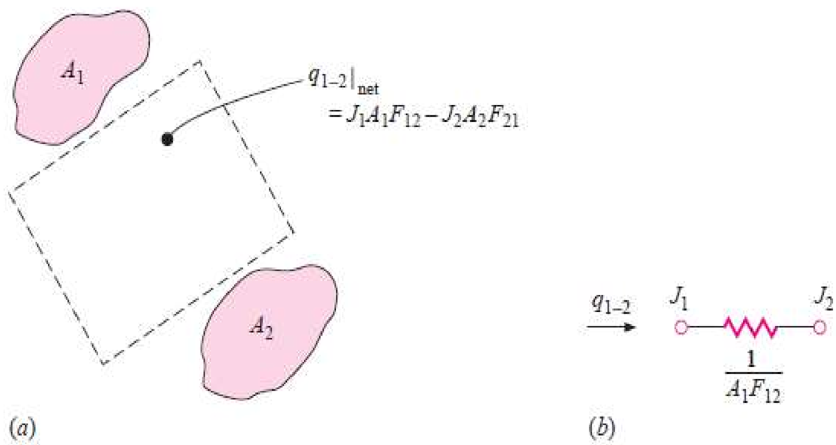
Now consider the exchange of radiant energy by two surfaces, A_1 and A_2, shown in Figure 8-25. Of that total radiation leaving surface 1, the amount that reaches surface 2 is

$$J_1 A_1 F_{12}$$

and of that total energy leaving surface 2, the amount that reaches surface 1 is

$$J_2 A_2 F_{21}$$

Figure 8-25 | (*a*) Spatial energy exchange between two surfaces; (*b*) element representing "space resistance" in the radiation-network method.



The net interchange between the two surfaces is

$$q_{1-2} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

But

$$A_1 F_{12} = A_2 F_{21}$$

so that

$$q_{1-2} = (J_1 - J_2) A_1 F_{12} = (J_1 - J_2) A_2 F_{21}$$

or

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}} \qquad \textbf{[8-39]}$$

$$q_{net} = \frac{E_{b1} - E_{b2}}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2}$$
$$= \frac{\sigma(T_1^4 - T_2^4)}{(1-\epsilon_1)/\epsilon_1 A_1 + 1/A_1 F_{12} + (1-\epsilon_2)/\epsilon_2 A_2} \qquad \textbf{[8-40]}$$

A network for a three-body problem is shown in Figure 8-27. In this case each of the bodies exchanges heat with the other two. The heat exchange between body 1 and body 2 would be

$$q_{1-2} = \frac{J_1 - J_2}{1/A_1 F_{12}}$$

and that between body 1 and body 3,

$$q_{1-3} = \frac{J_1 - J_3}{1/A_1 F_{13}}$$

To determine the heat flows in a problem of this type, the values of the radiosities must be calculated. This may be accomplished by performing standard methods of analysis used in dc circuit theory. The most convenient method is an application of Kirchhoff's current law to the circuit, which states that the sum of the currents entering a node is zero. Example 8-6 illustrates the use of the method for the three-body problem.

Figure 8-26 | Radiation network for two surfaces that see each other and nothing else.

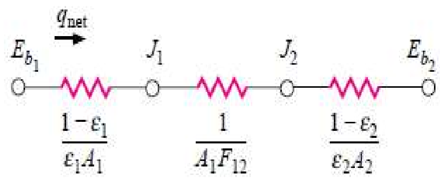
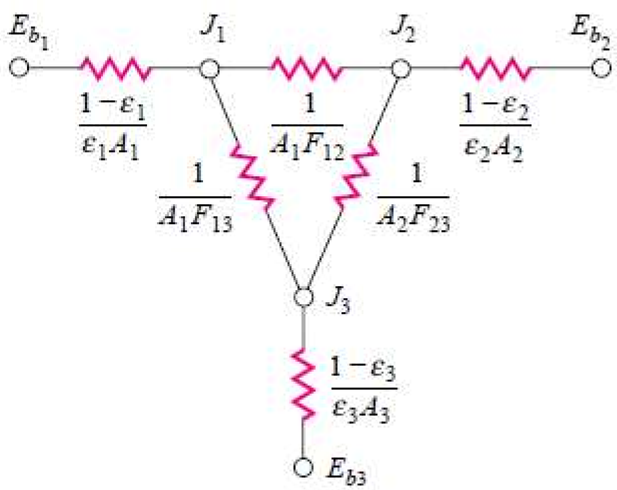
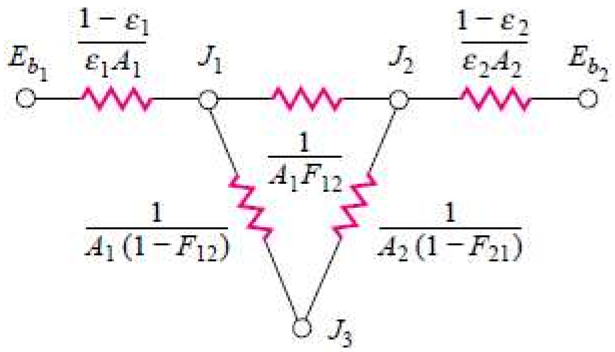


Figure 8-27 | Radiation network for three surfaces that see each other and nothing else.



Insulated Surfaces and Surfaces with Large Areas

Figure 8-28 | Radiation network for two plane or convex surfaces enclosed by a third surface that is nonconducting but re-radiating (insulated).



potential difference, and $J_3 = E_{b_3}$. Notice also that the values for the space resistances have been written

$$F_{13} = 1 - F_{12}$$
$$F_{23} = 1 - F_{21}$$

since surface 3 completely surrounds the other two surfaces. For the *special case where surfaces 1 and 2 are convex*, that is, they do not see themselves and $F_{11} = F_{22} = 0$, Figure 8-28 is a simple series-parallel network that may be solved for the heat flow as

$$q_{net} = \frac{\sigma A_1 (T_1^4 - T_2^4)}{\dfrac{A_1 + A_2 - 2A_1 F_{12}}{A_2 - A_1 (F_{12})^2} + \left(\dfrac{1}{\epsilon_1} - 1\right) + \dfrac{A_1}{A_2}\left(\dfrac{1}{\epsilon_2} - 1\right)} \qquad \textbf{[8-41]}$$

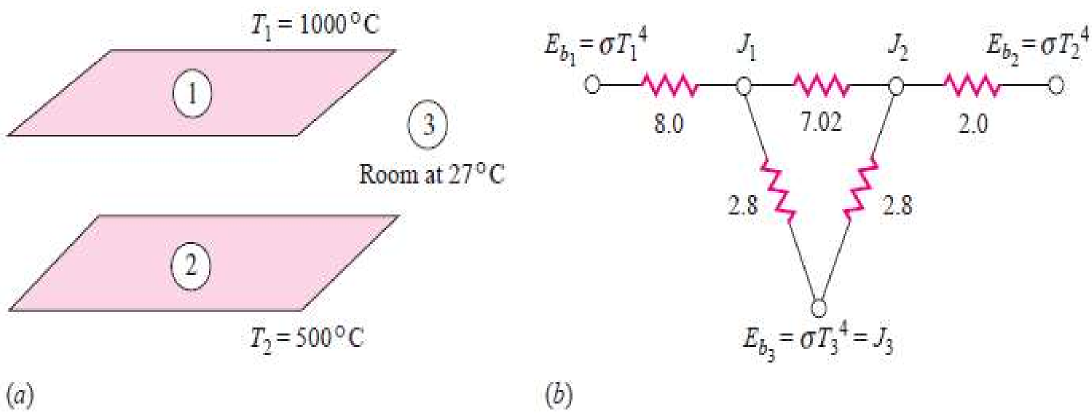
where the reciprocity relation

$$A_1 F_{12} = A_2 F_{21}$$

EXAMPLE 8-6 Hot Plates Enclosed by a Room

Two parallel plates 0.5 by 1.0 m are spaced 0.5 m apart, as shown in Figure Example 8-6. One plate is maintained at 1000°C and the other at 500°C. The emissivities of the plates are 0.2 and 0.5, respectively. The plates are located in a very large room, the walls of which are maintained at 27°C. The plates exchange heat with each other and with the room, but only the plate surfaces facing each other are to be considered in the analysis. Find the net transfer to each plate and to the room.

Figure Example 8-6 | (*a*) Schematic. (*b*) Network.



Solution

This is a three-body problem, the two plates and the room, so the radiation network is shown in Figure 8-27. From the data of the problem

$$T_1 = 1000°C = 1273 \text{ K} \qquad A_1 = A_2 = 0.5 \text{ m}^2$$
$$T_2 = 500°C = 773 \text{ K} \qquad \epsilon_1 = 0.2$$
$$T_3 = 27°C = 300 \text{ K} \qquad \epsilon_2 = 0.5$$

Because the area of the room A_3 is very large, the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ may be taken as zero and we obtain $E_{b_3} = J_3$. The shape factor F_{12} was given in Example 8-2:

$$F_{12} = 0.285 = F_{21}$$
$$F_{13} = 1 - F_{12} = 0.715$$
$$F_{23} = 1 - F_{21} = 0.715$$

The resistances in the network are calculated as

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{1-0.2}{(0.2)(0.5)} = 8.0 \qquad \frac{1-\epsilon_2}{\epsilon_2 A_2} = \frac{1-0.5}{(0.5)(0.5)} = 2.0$$
$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.5)(0.285)} = 7.018 \qquad \frac{1}{A_1 F_{13}} = \frac{1}{(0.5)(0.715)} = 2.797$$
$$\frac{1}{A_2 F_{23}} = \frac{1}{(0.5)(0.715)} = 2.797$$

Taking the resistance $(1-\epsilon_3)/\epsilon_3 A_3$ as zero, we have the network as shown. To calculate the heat flows at each surface we must determine the radiosities J_1 and J_2. The network is solved by setting the sum of the heat currents entering nodes J_1 and J_2 to zero:

node J_1:

$$\frac{E_{b_1} - J_1}{8.0} + \frac{J_2 - J_1}{7.018} + \frac{E_{b_3} - J_1}{2.797} = 0 \qquad [a]$$

node J_2:

$$\frac{J_1 - J_2}{7.018} + \frac{E_{b_3} - J_2}{2.797} + \frac{E_{b_2} - J_2}{2.0} = 0 \qquad [b]$$

Now

$$E_{b_1} = \sigma T_1^4 = 148.87 \text{ kW/m}^2 \quad [47,190 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_2} = \sigma T_2^4 = 20.241 \text{ kW/m}^2 \quad [6416 \text{ Btu/h} \cdot \text{ft}^2]$$
$$E_{b_3} = \sigma T_3^4 = 0.4592 \text{ kW/m}^2 \quad [145.6 \text{ Btu/h} \cdot \text{ft}^2]$$

Inserting the values of E_{b_1}, E_{b_2} and E_{b_3} into Equations (*a*) and (*b*), we have two equations and two unknowns J_1 and J_2 that may be solved simultaneously to give

$$J_1 = 33.469 \text{ kW/m}^2 \qquad J_2 = 15.054 \text{ kW/m}^2$$

The total heat lost by plate 1 is

$$q_1 = \frac{E_{b_1} - J_1}{(1-\epsilon_1)/\epsilon_1 A_1} = \frac{148.87 - 33.469}{8.0} = 14.425 \text{ kW}$$

and the total heat lost by plate 2 is

$$q_2 = \frac{E_{b_2} - J_2}{(1-\epsilon_2)/\epsilon_2 A_2} = \frac{20.241 - 15.054}{2.0} = 2.594 \text{ kW}$$

The total heat received by the room is

$$q_3 = \frac{J_1 - J_3}{1/A_1 F_{13}} + \frac{J_2 - J_3}{1/A_2 F_{23}}$$
$$= \frac{33.469 - 0.4592}{2.797} + \frac{15.054 - 0.4592}{2.797} = 17.020 \text{ kW} \quad [58,070 \text{ Btu/h}]$$

From an overall-balance standpoint we must have

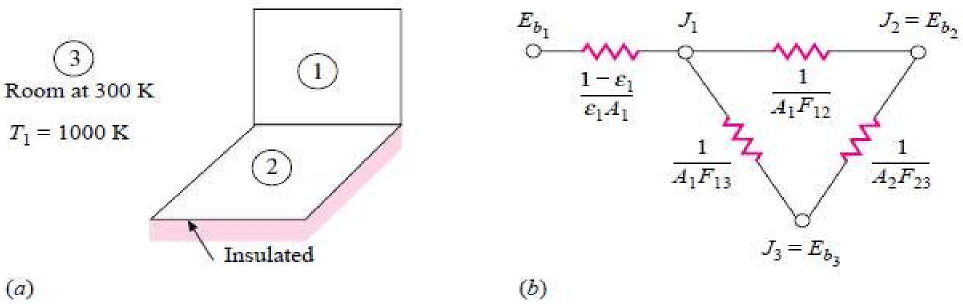
$$q_3 = q_1 + q_2$$

because the net energy lost by both plates must be absorbed by the room.

EXAMPLE 8-7 Surface in Radiant Balance

Two rectangles 50 by 50 cm are placed perpendicularly with a common edge. One surface has $T_1 = 1000$ K, $\epsilon_1 = 0.6$, while the other surface is insulated and in radiant balance with a large surrounding room at 300 K. Determine the temperature of the insulated surface and the heat lost by the surface at 1000 K.

Figure Example 8-7 | (*a*) Schematic. (*b*) Network.



Solution

Although this problem involves two surfaces that exchange heat and one that is insulated or re-radiating, Equation (8-41) may not be used for the calculation because one of the heat-exchanging surfaces (the room) is not convex. The radiation network is shown in Figure Example 8-7 where surface 3 is the room and surface 2 is the insulated surface. Note that $J_3 = E_{b_3}$ because the room is large and $(1 - \epsilon_3)/\epsilon_3 A_3$ approaches zero. Because surface 2 is insulated it has zero heat transfer and $J_2 = E_{b_2}$. J_2 "floats" in the network and is determined from the overall radiant balance. From Figure 8-14 the shape factors are

$$F_{12} = 0.2 = F_{21}$$

Because $F_{11} = 0$ and $F_{22} = 0$ we have

$$F_{12} + F_{13} = 1.0 \qquad \text{and} \qquad F_{13} = 1 - 0.2 = 0.8 = F_{23}$$

$$A_1 = A_2 = (0.5)^2 = 0.25 \text{ m}^2$$

The resistances are

$$\frac{1-\epsilon_1}{\epsilon_1 A_1} = \frac{0.4}{(0.6)(0.25)} = 2.667$$

$$\frac{1}{A_1 F_{13}} = \frac{1}{A_2 F_{23}} = \frac{1}{(0.25)(0.8)} = 5.0$$

$$\frac{1}{A_1 F_{12}} = \frac{1}{(0.25)(0.2)} = 20.0$$

We also have

$$E_{b_1} = (5.669 \times 10^{-8})(1000)^4 = 5.669 \times 10^4 \text{ W/m}^2$$

$$J_3 = E_{b_3} = (5.669 \times 10^{-8})(300)^4 = 459.2 \text{ W/m}^2$$

The overall circuit is a series-parallel arrangement and the heat transfer is

$$q = \frac{E_{b_1} - E_{b_3}}{R_{\text{equiv}}}$$

We have

$$R_{\text{equiv}} = 2.667 + \frac{1}{\frac{1}{5} + 1/(20+5)} = 6.833$$

and

$$q = \frac{56{,}690 - 459.2}{6.833} = 8.229 \text{ kW} \quad [28{,}086 \text{ Btu/h}]$$

This heat transfer can also be written

$$q = \frac{E_{b_1} - J_1}{(1-\epsilon_1)/\epsilon_1 A_1}$$

Inserting the values we obtain

$$J_1 = 34{,}745 \text{ W/m}^2$$

The value of J_2 is determined from proportioning the resistances between J_1 and J_3, so that

$$\frac{J_1 - J_2}{20} = \frac{J_1 - J_3}{20+5}$$

and

$$J_2 = 7316 = E_{b_2} = \sigma T_2^4$$

Finally, we obtain the temperature of the insulated surface as

$$T_2 = \left(\frac{7316}{5.669 \times 10^{-8}}\right)^{1/4} = 599.4 \text{ K} \quad [619^\circ\text{F}]$$

Comment

Note, once again, that we have made use of the $J = E_b$ relation in two instances in this example, but for two different reasons. $J_2 = E_{b_2}$ because surface 2 is insulated and there is zero current flow through the surface resistance, while $J_3 = E_{b_3}$ because the surface resistance for surface 3 approaches zero as $A_3 \to \infty$.